Pricing counterparty risk for American and Bermudian-type derivatives

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Counterparty risk and CVA

- Counterparty risk
  - Risk that a counterparty will default on her obligations in an OTC contract
- Basel III Credit Value Adjustment (CVA)
  - Adjustment to the derivative value
  - Measure of the expected potential loss from counterparty default
- Wrong Way Risk (WWR)
  - additional risk faced when the underlying asset and the default of the counterparty are correlated

Take-home message

- CVA valuation is easy for European derivatives, more complex for derivatives with early exercise opportunities
- Presently, computationnaly intensive Monte-Carlo methods are used for such cases
- We propose an efficient alternative that can be used for a large class of low-dimensional products
- The method yields the CVA values over the life of the contract, in a single pass

Outline

- Default model
- CVA computation
- Numerical experiments
- Wrong-Way Risk

Default model

- Simple intensity model: the time of default is the first jump date of a Poisson process
- Conditional on no prior default, probability at t of default before T is given by $1 - e^{-\lambda(T-t)}$
- Parameters can be easily inferred from market data (e.g. counterparty's CDS premium)
- Intensity λ can be time-dependent (deterministic or stochastic)

The European option case

- $CVA_t = V_t\left(1 - e^{-\lambda(T-t)}\right)$

- $V_t^D = e^{-\lambda(T-t)} V_t$

  Additionnal discount

where V_t is the price of a European option offered by a default-free counterparty with the same characteristics as the vulnerable option

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Early exercise opportunities

- Dates of the cash flow are not known
  - exercise strategy
  - underlying asset price

- Exposure falls to 0 after exercise

- Evaluation of the CVA by simulation
  - Find the exercise strategy and value function of the default-free option
  - Simulate asset price trajectories and default dates
  - Average the losses (or payoffs) on simulation paths

CVA computation: A Dynamic Programming approach

- M discrete exercise dates
- Value of a put option at m when the asset price is s

$$v_m(s) = \begin{cases} h_m(s) & \text{for } m = 0 \\ \max\{[K-s]^+, h_m(s)\} & \text{for } m = 1,\ldots,M-1 \\ [K-s]^+ & \text{for } m = M \end{cases}$$

Holding value

Decision

Exercise value

- Holding value at m when the asset price is s: expected value of the future potentialities of the contract

$$h_m(s) = \beta E_{m,s}\left[v_{m+1}\left(S_{t_{m+1}}\right)\right] \text{ for } m = 0,\ldots,M-1$$

Discount factor

CVA computation: a defaultable option

- Value of a defaultable put option at *m* when the asset price is *s*

$$v_m^D(s) = \begin{cases} h_m^D(s) & \text{for } m = 0 \\ \max\{[K-s]^+, h_m^D(s)\} & \text{for } m = 1,\ldots,M-1 \\ [K-s]^+ & \text{for } m = M \end{cases}$$

- Holding value is adjusted to account for the possibility of default

$$h_m^D(s) = \beta E_{m,s}\left[v_{m+1}^D\left(S_{t_{m+1}}\right)\right] - A_m(s)$$
$$h_m^D(s) = \beta' E_{m,s}\left[v_{m+1}^D\left(S_{t_{m+1}}\right)\right]$$

CVA computation

- The CVA can be obtaine
  by computing the differe

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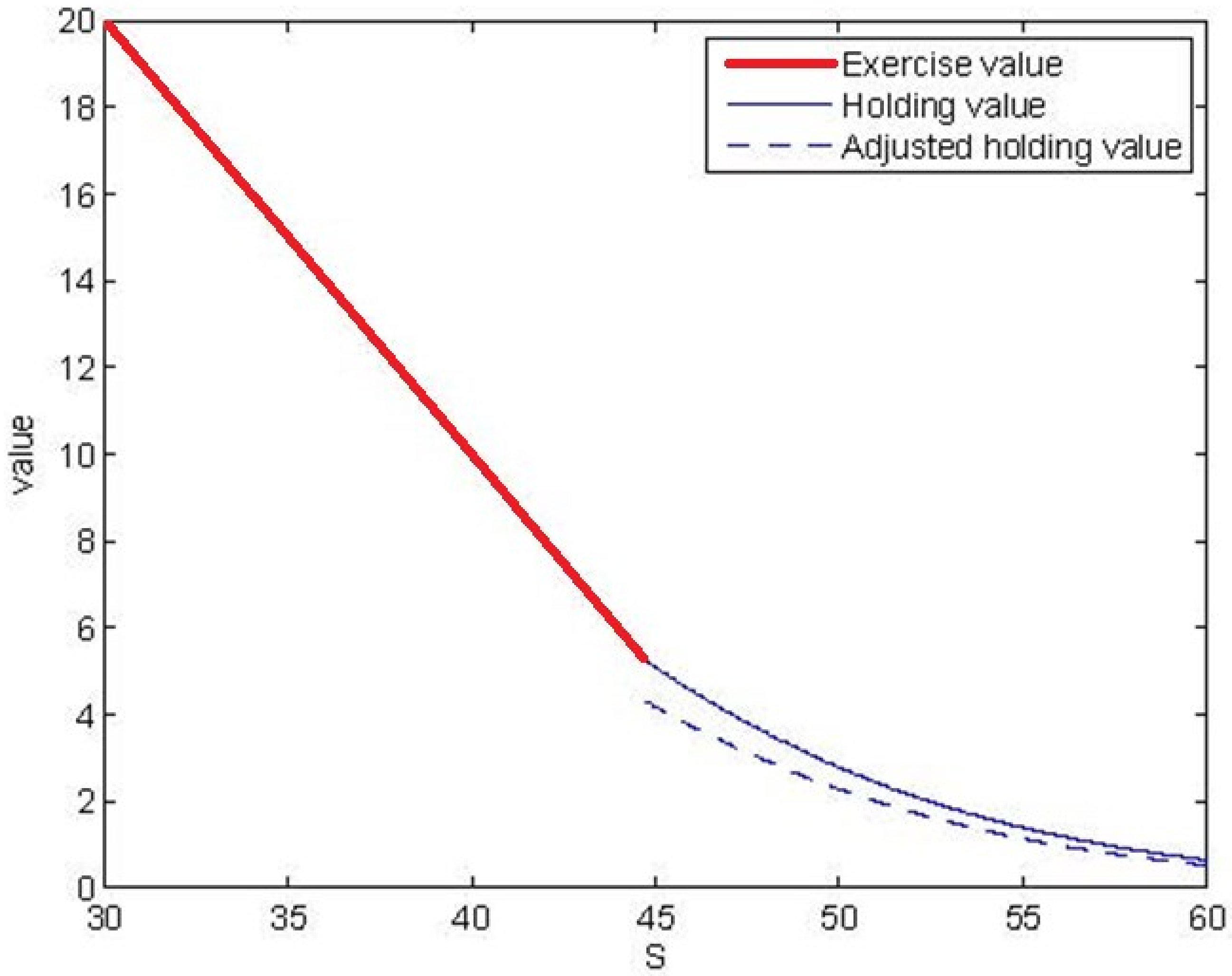
- Notice that the exercise
  corresponding counterp
  aware that she may not r

- If it were not the case, ar

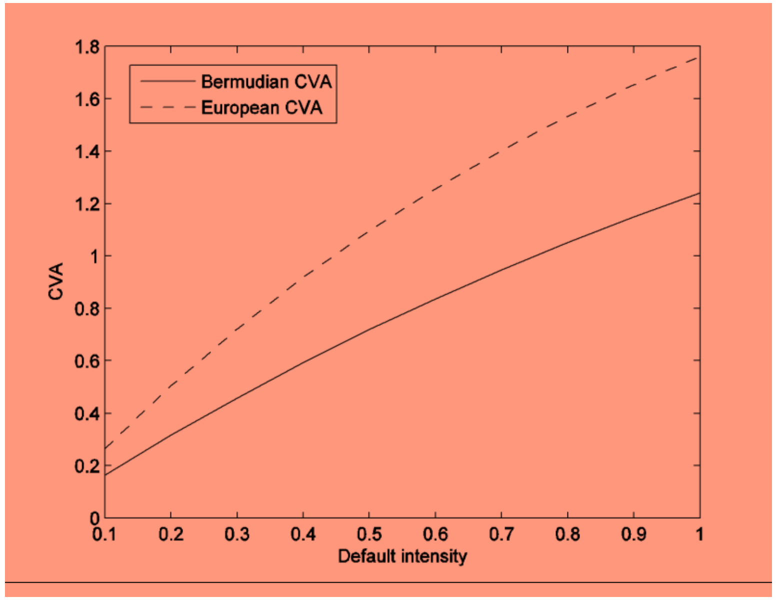


Numerical experiment: Lognormal model

- Bermudian put: $T = 1, M = 100, K = 60, r = 0.06, \sigma = 0.25$
- Simulation: 1 000 000 trajectories, 600 CPU seconds , precision 10^{-3}
- DP: 50 grid points, 0.5 CPU seconds, precision 10^{-5}

		DP	Simulation
	λ=0.1	4.3424	[4.3412,4.3455]
S_0=60	λ=0.3	3.8942	[3.8929,3.8998]
	λ=0.5	3.5002	[3.4976,3.5059]

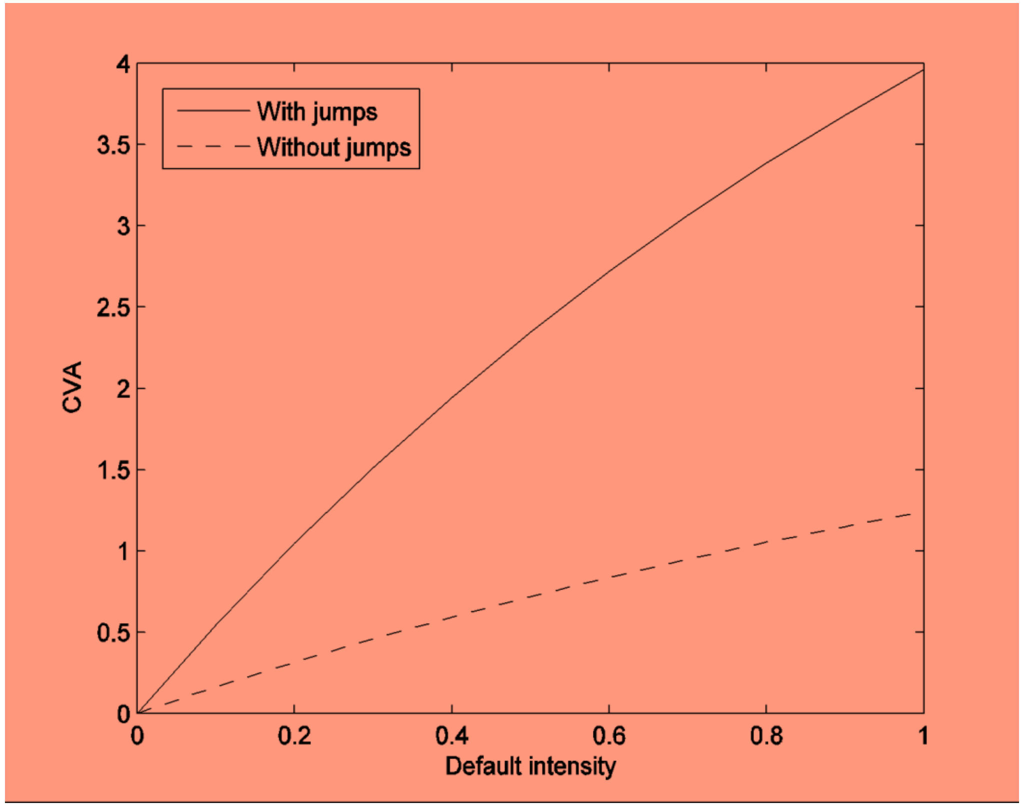
Adjusted prices



Bermudian vs European CVA

Numerical experiment: Jump diffusion model

- DP: 80 grid points, 8 CPU seconds, precision 10^{-3}
- The introduction of jump risk has a significant impact on the CVA, increasing with the default intensity.



CVA as a function of default intensity

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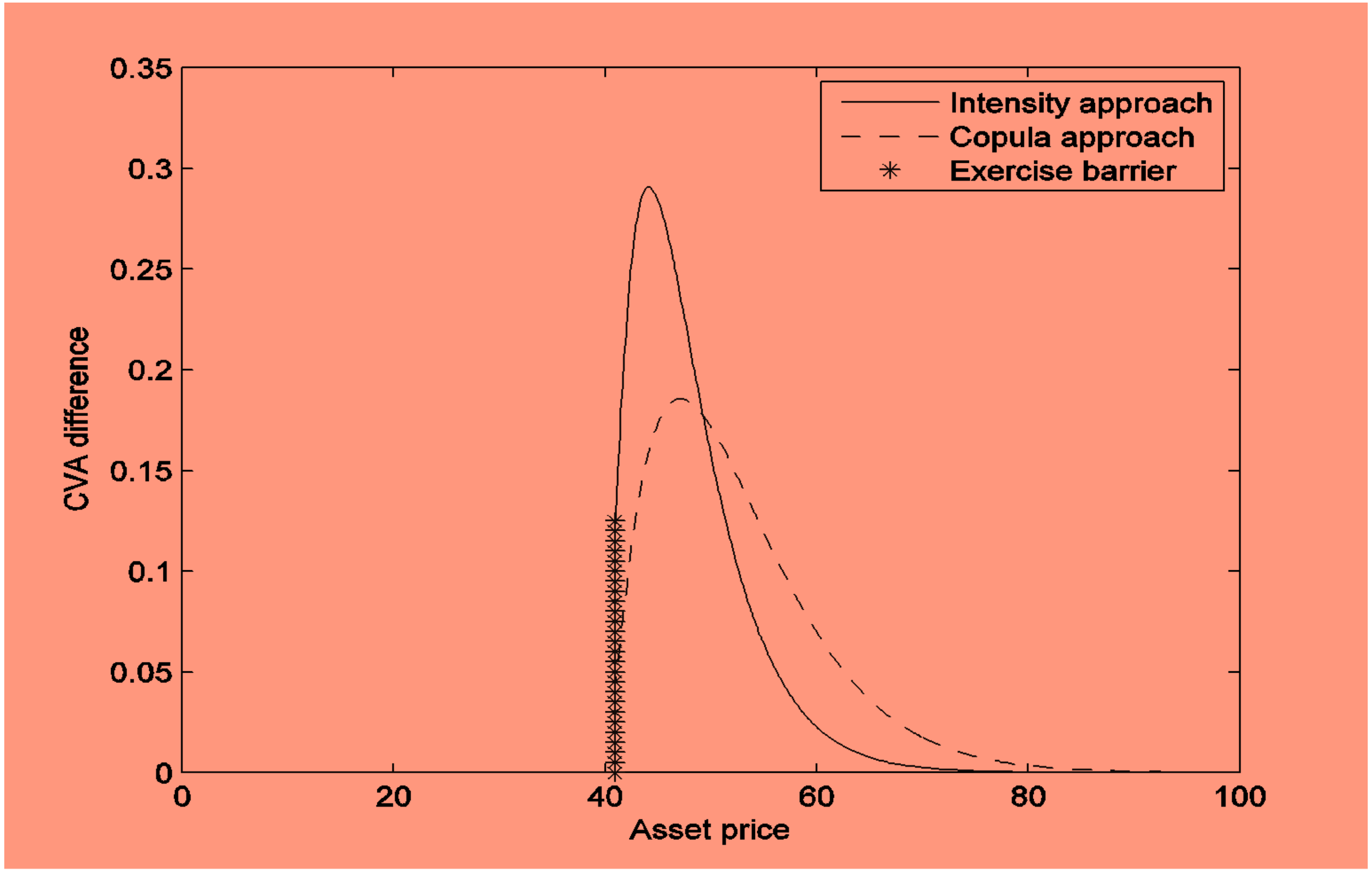
Wrong Way risk

- The additional risk when the underlying portfolio and the default time of the counterparty are correlated in the "wrong way"
- The presence of WWR increases the CVA (e.g. put options and positive correlation). Right-way risk has less impact
- Accounting for WWR is straightforward in both simulation-based and DP-based approaches
  - link the time evolution of the underlying asset with the default time (e.g. copulas)
  - ink the intensity of default to the asset price

Numerical experiment: lognormal model with WWR



Conclusion

- A very efficient approach to price options with early exercise opportunities subject to counterparty risk
- The DP can accommodate any market model, provided density functions can be evaluated efficiently, and any payoff function
- Unlike simulation-based methods, DP does not provide a point estimate, but the CVA as a function of time to maturity and asset price
- The framework is flexible and can easily account for counterparty risk features (e.g. WWR and collateral)